

# Gravitational Lensing And Extra Dimensions

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## Abstract

We study gravitational lensing and the bending of light in low energy scale ( $M_S$ ) gravity theories with extra space-time dimensions  $n$ . We find that due to the presence of spin-2 Kaluza-Klein states from compactification, a correction to the deflection angle with a strong quadratic dependence on the photon energy is introduced. No deviation from the Einstein General Relativity prediction for the deflection angle for photons grazing the Sun in the visible band with 15% accuracy (90% c.l.) implies that the scale  $M_S$  has to be larger than  $1.4(2/(n-2))^{1/4}$  TeV and approximately 4 TeV for  $n=2$ . This lower bound is comparable with that from collider physics constraints. Gravitational lensing experiments with higher energy photons can provide stronger constraints.

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Gravitational lensing or the gravitational bending of light, is one of the most important evidence which supports the Einstein General Relativity (EGR) theory [1]. Light sources are deflected when passing by a massive object. In EGR theory at grazing incidence the deflection angle is predicted to be  $\theta = 4G_N m/R$ , where  $m$  is the mass and  $R$  the radius of the massive object. For the Sun the deflection angle is  $1.75''$ . This prediction provides an important test for different theories of gravity [2–5]. In fact the detection of deflection angle of light passing by the Sun in 1919 was one of the most important first experiments which supported EGR theory [6]. Since then many other experiments have been carried out and found no deviation from the EGR theory [7–10]. It is usual to measure deviation from the EGR theory in terms of the post-Newtonian parameter  $\gamma$  defined by  $\theta = (4G_N m/R)(1+\gamma)/2$  which is one in EGR theory. The EGR theory is in agreement within a level better than one percent with experiments in the radio band to visible band [7,8].

There are other alternative theories for gravity, such as tensor-scalar theories [3] or theories with extra dimensions [4,5]. It is important to establish to what extent these theories are consistent with experiments in order to find the ultimate theory of gravity. In these alternative theories due to different type of gravitational interaction or new interactions in addition to the standard EGR interaction, there will be corrections to the parameter  $\gamma$ . Experimental measurements thus can provide strong constraints for other theories or even rule out some theories [2,3]. In this paper we study gravitational lensing in theories with extra space-time dimensions.

It has recently been proposed that gravitational effects can become large at a scale  $M_S$  near the weak scale due to effects from extra dimensions [4,5], which is quite different from the traditional concept that gravitational effects only become large at the Planck scale  $M_{Pl} = \sqrt{1/G_N} \sim 10^{19}$  GeV. In this proposal the total space-time has  $D = 4 + n$  dimensions. The relation between the scale  $M_S$  and the Planck scale  $M_{Pl}$ , assuming all extra dimensions are compactified with the same size  $R$ , is given by  $M_{Pl}^2 \sim R^n M_S^{2+n}$ . For  $n \geq 2$ ,  $M_S$  can be of order one TeV and  $R$  can be in the sub-millimeter region [4]. When the extra dimensions are compactified there are towers of states, the Kaluza-Klein (KK) states

with spin-2, spin-1 and spin-0, which interact with ordinary matter fields. There are many interesting consequences for collider physics [11,12], astrophysics and cosmology [13]. These new interactions provide information about the allowed value for  $M_S$ . The lower bound for  $M_S$  is constrained, typically, to be of order one TeV from collider experimental data [11,12]. There are also constraints from cosmological and astrophysical considerations [13].

Gravitational lensing is due to exchange of a massless graviton between photons and massive objects in EGR theory. In theories with extra dimensions gravitational lensing also receives contributions from the massive KK states in addition to that from the usual one. The massive KK states couple to matter fields in a way similar to the massless graviton. This makes gravitational lensing a sensitive test of theories with extra dimensions. We indeed find that the effects from massive KK states are significant and a term strongly dependent on the photon energy is introduced in the expression for the deflection angle if the scale  $M_S$  is in the TeV region. Experimental data on gravitational lensing by the Sun can provide interesting bounds on the scale  $M_S$  for these theories. The observation that there is no deviation from the ERG prediction with  $\gamma - 1 < 15\%$  (90% confidence level) in the range of visible light for light at grazing incidence to the Sun requires  $M_S$  to be larger than  $1.4(2/(n-2))^{1/4}$  TeV and approximately 4 TeV for  $n=2$ . This bound is comparable to that from collider physics experiments [11,12]. Gravitational lensing experiments with higher energies are able to put even more stringent limits on  $M_S$ . For a  $\gamma$ -ray of energy one MeV, no observed deviation from ERG at the 10% level would set a lower limit of  $1.5 \times 10^3$  TeV for  $M_S$ .

After compactifying the extra  $n$  dimensions, for a given KK level  $\vec{l}$  there are one spin-2,  $n-1$  spin-1 and  $n(n-1)/2$  spin-0 states [12]. Assuming that all standard fields are confined to a four dimensional world-volume and gravitation is minimally coupled to standard fields, it was found that the spin-1 KK states decouple while the spin-2 and spin-0 KK states couple to all standard fields [12]. We, however, found that only spin-2 KK states can interact with both the photon and the Sun. The graviton and the spin-2 KK states couple to the energy momentum tensor of the Sun which is similar to the coupling of a spin-2 particle to a scalar. There are different ways to obtain the deflection angle of light by a massive object. We will

treat the Sun as a scalar S and obtain the deflection angle by matching the scattering cross section and the impact parameter. The process studied is similar to photon-Higgs scattering [14]. The Feynman diagram is shown in Figure 1. Using the Feynman rules given in Ref. [12], we obtain the scattering amplitude for,  $\gamma(\epsilon_1(p_1)) + S(k_1) \rightarrow \gamma(\epsilon_2(p_2)) + S(k_2)$ , as

$$\begin{aligned}
M &= -4\pi G_N (m^2 \eta^{\mu\nu} + C^{\mu\nu,\rho\sigma} k_{1\rho} k_{2\sigma}) \left( \frac{B_{\mu\nu,\alpha\beta}^{graviton}}{q^2} + \sum_l \frac{B_{\mu\nu,\alpha\beta}^{KK}}{q^2 - m_l^2} \right) \\
&\quad \times (p_1 \cdot p_2 C^{\alpha\beta,\delta\gamma} + D^{\alpha\beta,\delta\gamma}) \epsilon_{1\delta}(p_1) \epsilon_{2\gamma}^*(p_2), \\
C^{\mu\nu,\rho\sigma} &= \eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \eta^{\mu\nu} \eta^{\rho\sigma}, \\
D^{\alpha\beta,\delta\gamma} &= \eta^{\alpha\beta} k_1^\delta k_2^\gamma - [\eta^{\alpha\gamma} k_1^\beta k_2^\delta + \eta^{\alpha\delta} k_1^\gamma k_2^\beta - \eta^{\delta\gamma} k_1^\alpha k_2^\beta + (\alpha \rightarrow \beta, \beta \rightarrow \alpha)], \\
B_{\mu\nu,\alpha\beta}^{graviton} &= \eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\nu\alpha} \eta_{\mu\beta} - \eta_{\mu\nu} \eta_{\alpha\beta}, \\
B_{\mu\nu,\alpha\beta}^{KK} &= (\eta_{\mu\alpha} - \frac{q_\mu q_\alpha}{m_l^2})(\eta_{\nu\beta} - \frac{q_\nu q_\beta}{m_l^2}) + (\eta_{\nu\alpha} - \frac{q_\nu q_\alpha}{m_l^2})(\eta_{\mu\beta} - \frac{q_\mu q_\beta}{m_l^2}) \\
&\quad - \frac{2}{3}(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{m_l^2})(\eta_{\alpha\beta} - \frac{q_\alpha q_\beta}{m_l^2}). \tag{1}
\end{aligned}$$

where  $q^2 = (p_1 - p_2)^2$ ,  $m$  is the scalar mass, and  $m_l$  is the mass of KK state. The sum is over all possible massive KK states. The term proportional to  $B_{\mu\nu,\alpha\beta}^{graviton}$  is the contribution from EGR theory due to the massless graviton, and the term proportional to  $B_{\mu\nu,\alpha\beta}^{KK}$  is the contribution due to the KK states. Gauge invariance dictates that the contributions from terms in  $B_{\mu\nu,\alpha\beta}^{gravity}$  and  $B_{\mu\nu,\alpha\beta}^{KK}$  proportional to  $\eta_{\mu\nu} \eta_{\alpha\beta}$  and any term which has an uncontracted Lorentz index on  $q$  vanish. Due to this property, the total contribution is simply related the pure massless graviton one by replacing  $1/q^2$  by  $1/q^2 + \sum_l 1/(q^2 - m_l^2)$ . We have

$$\begin{aligned}
M &= -16G_N \left( \frac{1}{q^2} + \sum_l \frac{1}{q^2 - m_l^2} \right) (\epsilon_1 \cdot \epsilon_2^* [p_1 \cdot k_1 p_2 \cdot k_2 + p_2 \cdot k_1 p_1 \cdot k_2 - p_1 \cdot p_2 k_1 \cdot k_2] \\
&\quad + p_1 \cdot p_2 [\epsilon_1 \cdot k_1 \epsilon_2^* \cdot k_2 + \epsilon_1 \cdot k_2 \epsilon_2^* \cdot k_1] + k_1 \cdot k_2 \epsilon_1 \cdot p_2 \epsilon_2^* \cdot p_1 \\
&\quad - p_1 \cdot k_2 \epsilon_1 \cdot p_2 \epsilon_2^* \cdot k_1 - p_1 \cdot k_1 \epsilon_1 \cdot p_2 \epsilon_2^* \cdot k_2 - p_2 \cdot k_2 \epsilon_1 \cdot k_1 \epsilon_2^* \cdot p_1 - p_2 \cdot k_1 \epsilon_1 \cdot k_2 \epsilon_2^* \cdot p_1). \tag{2}
\end{aligned}$$

For small deflection angles the photon energies  $\omega_1$  and  $\omega_2$  are approximately the same which will be indicated by  $\omega$ , and  $q^2 = -4\omega_1 \omega_1 \sin^2(\tilde{\theta}/2) \approx -\omega^2 \tilde{\theta}^2$ . Here  $\tilde{\theta}$  is the angle between the incoming and outgoing photon directions. Neglecting small terms proportional to  $\tilde{\theta}$  in the numerator, we obtain

$$\frac{d\sigma}{d\Omega} = 16G_N^2 m^2 \left( \frac{1}{q^2} + \sum_l \frac{1}{q^2 - m_l^2} \right)^2. \quad (3)$$

Without the massive KK contribution, the result reduces to the standard one.

All possible KK states have to be summed over. The masses for the KK states are given by  $m_l^2 = 4\pi^2 \vec{l}^2 / R^2$ , where  $\vec{l}$  represents the hyper-cubic lattice sites in n-dimensions. For  $M_S$  in the multi-TeV range the KK states are nearly degenerate and the sum can be approximated by integral in n-dimensions. Using the result in Ref. [12], we obtain

$$\Delta = \sum_l \frac{1}{q^2 - m_l^2} = -\frac{2}{M_S^4 G_N} \left( \frac{|q^2|}{M_S^2} \right)^{n/2-1} I_n(M_S / \sqrt{|q^2|}), \quad (4)$$

with

$$I_n = \int_{M_{min}/\sqrt{|q^2|}}^{M_S/\sqrt{|q^2|}} \frac{y^{n-1}}{1+y^2} dy, \quad (5)$$

where  $M_{min}$  is the minimal KK state mass  $2\pi/R$  which is of order  $10^{-3}$  eV for  $R$  in the milli-meter range. The leading contribution to  $\Delta$  for  $|q^2|/M_S^2 \ll 1$ , which is true in our case, is equal to  $(1/(M_S^4 G_N)) \ln(M_S^2/(|q^2| + M_{min}^2))$  for  $n = 2$ , and  $(1/(M_S^4 G_N))(2/(n-2))$  for  $n > 2$ . In the above we have used  $G_N = (4\pi)^{n/2} \Gamma(n/2) R^{-n} M_S^{-(n+2)}$ .

The differential cross section for small scattering angle  $\tilde{\theta}$  can be written as

$$\frac{d\sigma}{d\Omega} = 16G_N^2 m^2 \left( \frac{1}{\tilde{\theta}^2} + \omega^2 \Delta \right)^2, \quad (6)$$

Keeping the leading correction to the deflection angle  $\theta$ , we obtain

$$\theta = \frac{4G_N m}{R} \left( 1 - 2\omega^2 \Delta \left( \frac{4G_N m}{R} \right)^2 \ln \left( \frac{4G_N m}{R} \right) \right). \quad (7)$$

We note that effect of extra dimensions is always to increase the deflection angle, and also to introduce a  $\omega$  dependence in the deflection angle. In the EGR theory an  $\omega$  dependence can be generated at one loop order [15]. However, there the contribution is extremely small. The contribution from extra dimensions obtained here can be very large—close to the present experimental reach. For easy comparison with data we work with the post-Newtonian parameter  $\gamma$ . The expression for  $\theta$  gives the correction  $\Delta\gamma = \gamma - 1$  as

$$\Delta\gamma = -4\omega^2\Delta\left(\frac{4G_N m}{R}\right)^2 \ln\left(\frac{4G_N m}{R}\right). \quad (8)$$

For the Sun  $R_\odot = 6.96 \times 10^5$  km, and  $2G_N m_\odot = 2.95$  km, we obtain the correction to  $\gamma$  for grazing deflection of light by the Sun

$$\Delta\gamma = -0.50 \left(\frac{\omega^2}{\text{eV}^2}\right) \left(\frac{1\text{TeV}}{M_S}\right)^4 \delta, \quad (9)$$

with  $\delta = 2/(n-2)$  for  $n > 2$ , and  $\delta \approx \ln(M_S^2/(\omega^2\theta^2 + m_{min}^2))$  for  $n = 2$ . For  $n=2$ ,  $\delta$  is of order 50 to 80 for  $\omega$  in the range of radio waves to  $\gamma$ -rays and  $M_S$  in the range of multi-TeV. Using the above one can extract important information about theories with extra dimensions.

Our calculations correspond to the determination of the total deflection of the light coming from a distant source and grazing the Sun, so that the impact parameter is  $R_\odot$ . This can easily be generalized to the total deflection with an arbitrary impact parameter  $b$ , simply by replacing  $R_\odot$  by  $b$ . As the correction induced by the KK states to the EGR is small, we may assume that the photon follows a post Newtonian geodesic path to obtain the deflection angle  $\delta\alpha$  measured at the Earth [16] with

$$\delta\alpha = \frac{(1+\gamma)G_N m_\odot}{r_E} \frac{\sin\alpha}{1-\cos\alpha}, \quad (10)$$

where  $\alpha$  is the angle between the direction of Earth-to-Sun and the incoming light ray to the detector on the Earth, and  $r_E$  is the Earth-Sun distance. The impact parameter is  $b = r_E \sin\alpha$ . In our case, the parameter  $\gamma$  is not a constant. It is given by

$$\gamma = 1 - 4\omega^2\Delta\left(\frac{4G_N m_\odot}{r_E \sin\alpha}\right)^2 \ln\left(\frac{4G_N m_\odot}{r_E \sin\alpha}\right), \quad (11)$$

and depends not only on  $\omega^2$ , but also on the angle  $\alpha$ .

Experimental observations have found no deviations from the EGR theory prediction for  $\gamma$  from radio waves to visible light. For  $M_S = 1$  TeV, the typical limit set by most of the collider experiments, there is no conflict for photons with frequencies below the visible. Experimental observations of gravitational lensing by the Sun in visible light from whole sky survey of Hipparcos have found [8]  $\gamma = 0.997 \pm 0.003$ . This is a very impressive result.

Unfortunately this value can not be used directly in our case because in the Hipparcos analysis,  $\gamma$  was assumed to be constant in the whole range of  $\omega$  and  $b$  and most of the data was at large  $b \geq r_E/2$ . In our case the largest deviation from EGR is reached for light grazing the Sun. In this region the accuracy of the observations is not as good as the whole sky result. The result for visible light near the solar limb is  $\gamma = 0.95 \pm 0.11$  [10], which is considerably less accurately measured. However even with such accuracy, we find that the mass  $M_S$  is constrained to be larger than about  $1.4(2/(n-2))^{1/4}$  TeV at  $2\sigma$  level for  $n > 2$  and a factor of approximately 3 larger for  $n = 2$ . This bound is comparable with the limit obtained from collider data. Radio data from sources near the Sun give  $\gamma = 1.001 \pm 0.002$  [9], consistent with 1 as we would expect for very low frequency photons. With  $\gamma$ -rays of energy one MeV, no observed deviation from EGR theory up to 10% would imply that the scale  $M_S$  must be larger than  $1.5 \times 10^3$  TeV which is much stronger than any collider experimental bounds.

We suggest that future studies of the parameter  $\gamma$  should vigorously investigate its frequency dependence and its impact parameter dependence. Theories of the type considered here, with mass  $M_S$  about 3 TeV scale, suggest  $\gamma - 1$  is negligible for radio frequencies, is positive of order  $3 \times 10^{-3}$  in the visible and is so large at  $\gamma$ -ray frequencies that our approximation are no longer valid for light grazing the Sun. For larger impact parameters, the effect can become much smaller.

The same analysis can be carried out for other systems. Due to smaller ratios of mass to radius for the planets in the solar system, the corrections for the gravitational lensing by planets in the solar system are small beyond the reach for near future experimnts. However, gravitational lensing by heavier objects, such as quasars with known masses and radii the effects of the extra dimensions can be large. Precision experiments on gravitational lensing for these objects can provide important information about the theory of gravity and possible extra dimensions.

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# FIGURES

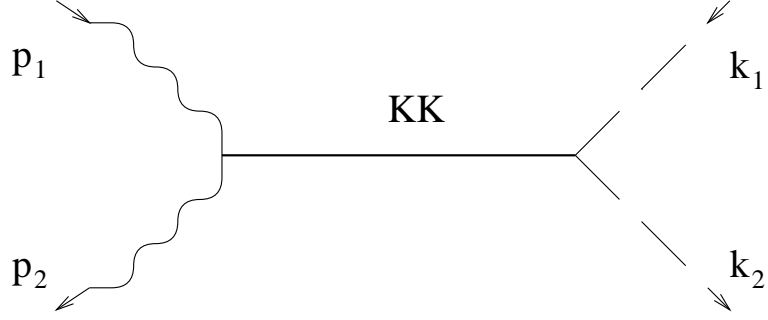


FIG. 1. The Feynman diagram for KK states contribution to  $\gamma(p_1)\gamma(p_2) \rightarrow S(k_1)S(k_2)$ .